



Interpretable Embeddings for Representing Categorical Variables within Discrete Choice Models

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Outline

- Motivation
- Embeddings representations
- Model Formulation & Network Architecture
- Experiment: Swissmetro Dataset Mode choice
- Results and Comparison to Benchmark models
- Visualizing and Interpreting the embeddings representations
- Conclusions and Future Directions



Motivation: issues with dummy encoding

(1) Conceptual challenges

Do not reflect human perception and intuition Orthogonal and equidistant representations

e.g.

Self-Employed, Employee, Unemployed, Student, Pupil

Self-Employed, Employee, Unemployed, Student, Pupil

(2) Challenges related to data and estimation requirements

Including richer data \rightarrow crucial for promoting transport research (Ben-Akiva et al., 2002)

• Modeling perspective:

Model's complexity increases proportionally to the cardinality of the variables \rightarrow increased sample size requirements to avoid overfitting or poor parameter estimation

(Rose et al., 2013)(Giron et al., 2010)(Cherchi et al., 2012)

• Survey design perspective:

Travel data collection \rightarrow most expensive and time-consuming part of the model development process (Kulpa,& Szarata, 2016)

Data-driven (NN) approach for encoding categorical variables:

- More intuitive for human analyzer
- More compact yet informative representations of the data

Embeddings encoding



Embeddings representations



- Latent *lower space representations* (vectors) of the input data
- Their weights are learned in a supervised manner using a (shallow) NN architecture
- Performing *dimensionality reduction* while preserving the *relevant information to a given predictive task*

5



Travel behavior embeddings

• Previous work:

Pereira, F. C. (2019). Rethinking travel behavior modeling representations through embeddings. Sifringer, B., Lurkin, V., & Alahi, A. (2020). Enhancing discrete choice models with representation learning.

- Current work: Embeddings Multinomial Logit (E-MNL) model
 - ➤ NN-based DCM
 - > Embeddings for encoding sociodemographic and travel variables

Compared with previous work:

- 1. estimate jointly the embedding weights and the parameters (betas) of the DCM
- 2. all categories across variables embedded in the same continuous space
- 3. constraints for enforcing interpretability \rightarrow model with fully interpretable params



E-MNL Model Formulation

We consider a choice set C with J number of alternatives,

 $X = \{X_1, X_2, \dots, X_K\} \rightarrow \text{continuous}$ $Q = \{Q_1, Q_2, \dots, Q_M\} \rightarrow \text{categorical}$

such that the systematic part of the utility of that individual *n* associates with alternative i = 1, ..., J, is given by:

 $V_{i,n}(X_n, Q_n) = f(X_{i,n}; \boldsymbol{B}) + f(g_i(Q_n; \boldsymbol{W}); \boldsymbol{B}')$

B, *B*': trainable preference parameters (betas)

W: trainable weights of the embedding matrix (values of embedding vectors)

g: embedding function \rightarrow mapping each category in Q_n to an embedding representation Q'_n , such that:

 $g(Q_n; \boldsymbol{W}) = Q'_n$

f: linear function for the utilities such that: $f(X_{i,n}; B) = BX_{i,n}$ and $f(g(Q_n; W_i); B') = B'Q'_{i,n}$

$$V_{i,n}(X_n, Q_n) = \boldsymbol{B} X_{i,n} + \boldsymbol{B}' Q'_{i,n}$$



E-MNL– model architecture



8



Swissmetro Dataset - Predicting Mode Choice

- Swissmetro dataset (Bierlaire et al., 2001): openly available dataset for mode choice / SP data.
- Three alternatives (J=3): Train, Swissmetro, Car
- 9,036 obs \rightarrow split into training (7,234 obs) and test set (1,802 obs)

Variable	Description	Туре
PURPOSE	Trip purpose (business, leisure etc.)	Categorical
FIRST	First class traveler	Binary
TICKET	ticket type (one-way, half-day, etc.)	Categorical
WHO	Who pays (self, employer etc.)	Categorical
MALE	Traveler's gender	Binary
INCOME	Traveler's income level per year	Ordinal
LUGGAGE	Traveler's luggage pcs	Ordinal
AGE	Traveler's age class	Ordinal
SEATS	Airline Seat configuration in the Swissmetro	Binary
GA	Annual season ticket	Binary
ORIGIN	Travel origin corresponding to a canton	Categorical
DEST	Travel destination corresponding to a canton	Categorical
TT	Door-to-door travel time in minutes, scaled by 1/100	Continuous
TC	Travel cost in CHF, scaled by 1/100	Continous
HEADWAY	Transportation headway in minutes, scaled by 1/100	Continous

Benchmark Models	Specification
MNLorig	original utility specification from (Bierlaire et al., 2001)
MNLdum	all variables / categorical as dummies
MNL dum - (NO OD)	same as above but excluding ORIGIN and DEST

E-MNL \rightarrow all variables / categorical as embeddings

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Results – Comparison to Benchmark Models

				Model	Params	Estimates	Std errors	t-stat	p-value
Suggested Model	$\begin{array}{ c c }\hline LL_{train} \\ (std.) \end{array}$	$\frac{\overline{LL}_{test}}{(std.)}$	AIC (std.)	E-MNL LL _{train} : -4911.4	ASC_{Car} ASC_{SM}	0.252 0.476	0.093 0.079	2.722 6.022	0.006 0.000
E-MNL	-4912.7 (3.0)	- 1231.1 (1.1)	10345.5 (6.0)	LL_{test}^{num} : -1229.2	TT TC HEAD _{Train,SM} PURPOSE	-1.536 -1.129 -0.808 14.074	0.054 0.043 0.122 1.052	-28.437 -26.099 -6.647 13.377	0.000 0.000 0.000 0.000
Benchmark Models	LL _{train}	LL _{test}	AIC		FIRST	2.059	1.142 0.676	1.802 19.167	0.072
MNL _{orig} MNL _{dum} (No OD) MNL _{dum}	-5764 -5243 - 4903	-1433 -1292 -1928	11546 10604 10066		WHO LUGGAGE AGE MALE INCOME GA ORIGIN DEST SEATS	8.256 7.118 8.484 3.752 7.417 7.429 15.325 15.425 7.526	1.392 2.428 0.894 2.078 1.124 1.479 1.949 1.572	5.929 2.931 9.494 1.806 6.599 5.022 7.862 9.814 4.662	$\begin{array}{c} 0.000\\ 0.003\\ 0.000\\ 0.071\\ 0.000\\ 0.$

Swissmetro embeddings: comparing categories within variables



Alternative-specific dimensions

	Train	Car	SM
.g. self	[0.018,	- 0.004	-0.005]

Signs (B' > 0) & Magnitude of each dimension

Embeddings for mode choice trained on the Swissmetro dataset.

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Swissmetro embeddings: comparing categories across variables



- All categories are embedded into the same continuous space
- Similar categories have similar vectors – Categories close in space
 - → related to the study behavior in a similar manner

• Formation of meaningful clusters

Embeddings for mode choice trained on the Swissmetro dataset.

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Conclusions

(1) Embedding representations \rightarrow behaviorally meaningful outputs \rightarrow diagnostic and analysis purposes

(2) Increased predictive performance

(3) Efficiently incorporating categorical variables with high cardinality without overfitting

(4) Preserving direct interpretability for all the model's parameters despite being based on NN principles:

➤Utilities coefficients → measuring contribution of all the input variables to the models' predictions

➤Alternative-specific embedding values → reflecting internal variable relationships / further insights into understanding the observed behavior



Future directions

- Extending E-MNL for Nested, Mixed, Latent Class Logit models
- Use embeddings for modeling the dynamics of behavior → how do these representations change across time and space?
- Estimating embeddings for multiple outputs Multitask Learning → generalized embeddings representations → describe travel-behavior as a whole?

Thank you!

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Find us: <u>https://mlsm.man.dtu.dk/</u>

Code: <u>https://github.com/ioaar/Interpretable-Embeddings-MNL</u>



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